Optimal placement of piezoelectric plates to control multimode vibrations of rotating beams

F. Botta[1*]- D. Dini[2]- S.Gentili[1]- G. Cerri[1]
[1] Dipartimento di Ingegneria Meccanica e Industriale, Università degli studi Roma Tre, Roma, Italy, [2] Department of Mechanical Engineering-Imperial College London - United Kingdom
*Corresponding author: Via della Vasca Navale, 79 – 00146 Rome, fabio.botta@uniroma3.it

Abstract: Vibrations of rotating beam, i.e. turbomachine blades, and the associated fatigue phenomena, are responsible of the reduction of their life and sometimes can give catastrophic failures. Typically passive damping systems are used but, differently from the active system, they are not able to change their characteristics depending on the system response. In the last years the use of the piezoelectric materials, for active damping, has received considerable attention by many researchers and most recently also for rotating beam. The effectiveness of such system strongly depends on their position relatively to the excited modes. In this paper we use the finite element code COMSOL to find the optimal position of piezoelectric plates to control the multimode vibrations of a rotating beam.

Keywords: piezoelectric material, vibrations, blades, turbomachine, optimal position

1. Introduction

Damping systems are often used to avoid problems connected with the vibrations of rotating beam. Blade vibrations in aircraft engines, for example, are typically damped by passive systems such as friction damping. In the last two decades, the adoption of piezoelectric elements, has received considerable attention by many researchers for its potential applicability to different areas of mechanical, aerospace, aeronautical and civil engineering. These structures show an interesting coupling between electrical and mechanical quantities: a deformation appears when an electric field is applied and vice versa. More recently studies about their use in rotating beam have been carried out, but only few of these concern with active damping ([7]-[10]). In contrast with passive damping elements, active elements are able to change their characteristics depending on the system response. Considering that the effectiveness of the piezoelectric elements to damp a particular excited mode, or a multimode combinations, strongly depends on their position, and that the excited modes change during the time, the possibility of an active system to change the work-configuration of the piezo-plates can increase considerably their efficiency. In the last years the study of their optimal position has received increasing attention [5]. Typically the aim of these studies is to find the position that minimizes an objective function or maximizes the degree of modal controllability. Barboni et al. [5] the possibility of exciting the flexural dynamics of an Euler-Bernouilli beam, according to a single mode, is examined. The results show that, to excite a desired mode, the actuator must be placed between two consecutive points at which the curvature becomes zero. Unfortunately in many real cases the loads applied to the structures excite more than one mode and with different amplitudes. Some of the authors have extended this procedure in ([2]) where a new function to find the optimal placement of piezoelectric plates to control the multimode vibrations of a beam, and an analytical solution, has been proposed. The results have been also compared with FEM simulations, experimental data ([1]) and results from the literature with very good agreement. In this paper FEM simulations are extended to a rotating beam with a bimodal excitation. Optimal configurations have been found for different angular velocities and different ratio between the two modes.

Table 1: Nomenclature

a	axis position of the centre of the piezo plates
A	transversal area of the beam
c	beam width
d_{31}	piezoelectric coefficient
E_a	Young's modulus of the piezoelectric material
E_b	Young's modulus of the beam
h	piezo plates length
I_b	inertia moment of the beam
L_b	beam length
M_a	piezoelectric bending moment
S	transversal area of the beam
T_a	piezoelectric thickness
T_b	beam thickness
u	axial displacement
V	voltage applied to the piezoelectric plates
w	vertical displacement
α	damping coefficient
β	damping coefficient
ρ	density
ω_i	natural frequency
Ω	angular velocity of the beam

2. Governing equation for piezoelectric rotating beam

In Fig. 1 a piezoelectric rotating beam is schematically represented; the two PZT plates are applied in a symmetrical position with respect to the medium plane. Indicating with δL_e, δL_{in}, δL_a the virtual work of the elastic, inertial and piezoelectric forces, respectively, the principle of the virtual works can be written as:

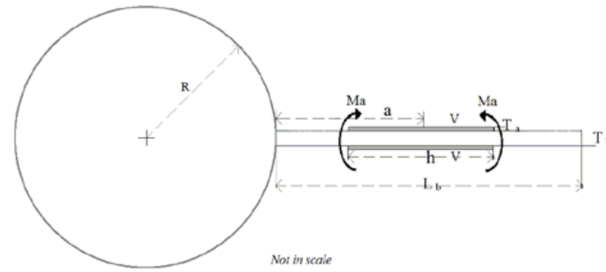

Figure 1. Rotating piezo-beam

$$\delta L_e + \delta L_{in} + \delta L_a = 0 \qquad (1)$$

The virtual work of the elastic and inertial forces is:

$$\begin{aligned}\delta L_e = {} & EA\int_0^{L_b}\left[\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]\frac{\partial \tilde{u}}{\partial x}dx + \\ & +EA\int_0^{L_b}\left[\frac{\partial u}{\partial x}\frac{\partial w}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^3\right]\frac{\partial \tilde{w}}{\partial x}dx + \\ & +EI\int_0^{L_b}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 \tilde{w}}{\partial x^2}dx;\end{aligned} \qquad (2)$$

$$\begin{aligned}\delta L_{in} = {} & \rho A\int_0^{L_b}\left[\Omega^2(R+x+u)-\ddot{u}+2\Omega\dot{w}\right]\tilde{u}\,dx + \\ & +\rho A\int_0^{L_b}\left[\Omega^2 w-\ddot{w}-2\Omega\dot{u}\right]\tilde{w}\,dx \\ & +\rho I\int_0^{L_b}\left[\Omega^2\frac{\partial w}{\partial x}-\frac{\partial \ddot{w}}{\partial x}\right]\frac{\partial \tilde{w}}{\partial x}dx.\end{aligned}$$

Using the PIN Force model [1], the action of the plate can be modelled by two flexural moments applied at the end of the plate (Fig. 1) with:

$$M_a(t)=\frac{\psi}{6+\psi}E_a c T_a T_b \Lambda(t) \qquad (3)$$

and:

$$\begin{cases} \Lambda(t) = \dfrac{d_{31}}{T_a} V(t) \\ \psi = \dfrac{E_b T_b}{E_a T_a} \end{cases} \tag{4}$$

so that the virtual work of the PZT plate will be:

$$\delta L_a = M_a \left(\left. \frac{\partial \tilde{w}}{\partial x} \right|_{x=a+\frac{h}{2}} - \left. \frac{\partial \tilde{w}}{\partial x} \right|_{x=a-\frac{h}{2}} \right) \tag{5}$$

The variables *a* and *h* can vary within the domain:

$$\begin{cases} 0 \le a - \dfrac{h}{2} \le L_b \\ 0 \le a + \dfrac{h}{2} \le L_b \\ 0 \le a \le L_b \\ 0 \le h \le L_b \end{cases} \tag{6}$$

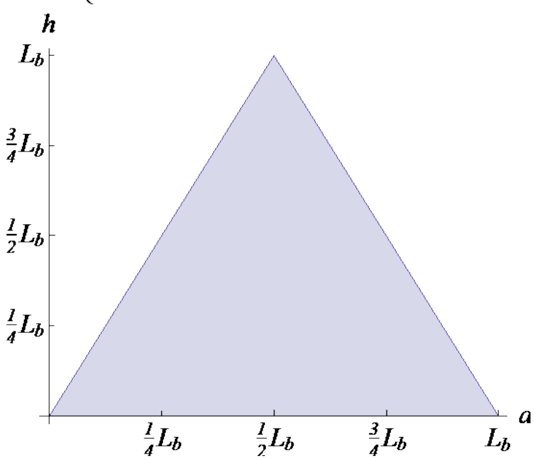

Figure 2. Domain of the variables a and h

and their value depends on the modes that must be damped. If a single mode excitation is considered, Barboni et al. ([5]) found the optimal placement of PZTs. This is, obviously, also the optimal placement to damp that mode's vibration if the PZT actuators are employed to apply a load in counter-phase to the external excitation. However in some practical applications, i.e. gas turbine blades, the response spectrum can include various modes, each of which is characterized by different amplitudes. Hence, a strategy to looking for the optimal placement for multimodal damping needs to be developed. In this paper the piezo-plate will be used to apply a multimode load excitation by the tension V(t) ((3), (4)). For every load condition the most effective (optimal) position, defined by a and h, will be that which maximizes the amplitude of the vertical displacement of the free end of the beam. This has been evaluated, for different angular velocities, by frequency response analysis and the solution for multi-modes loads has been achieved by the linearly combination of the single-mode vibrations. In all the simulations the Rayleigh damping is used:

$$\mathbf{C} = \alpha \mathbf{M} + \beta \mathbf{K}$$

where **M** and **K** are, respectively, the mass and the stiffness matrices.

3. Numerical results and discussion

This section reports results for bi-modal excitation of a rotating beam with the characteristics of Tab. 2.

Table 2: Characteristics of the rotating beam

Length [mm]	300
Width [mm]	30
Thickness [mm]	3
Hub [mm]	700
Young's modulus [GPa]	210
Mass volume density [Kg/m^3]	7700
Damping coefficient α	2.75
Damping coefficient β	0

Indicating with r the percentage of the excitation induced for the j-th mode, the expression of the tension V(t) is:

$$V(t) = (1-r)\cos(\omega_i t) + r\cos(\omega_j t) \tag{7}$$

When it is r=0, or r=1 just a single mode is excited and, indicating with $\varphi_i(x)$ the i-th eigenmode, the virtual work of the PZT plate is:

$$\delta L_a = M_a\left(\tilde{\varphi}_i^I\left(a+\frac{h}{2}\right)-\tilde{\varphi}_i^I\left(a-\frac{h}{2}\right)\right) \quad (8)$$

The optimal configuration is that one that maximizes the difference $\varphi_i^I(a+h/2)$- $\varphi_i^I(a-h/2)$, then the optimal placement of the PZT will be obtained when the ends of the plates (a+h/2, a-h/2) coincide with the abscissa of the absolute maximum and minimum of $\varphi_i^I(x)$.

Because of the absolute maximum of $|\varphi_i^I(x)|$ is always in $x=L_b$, independently from the excited modes and for any angular velocity Ω, hence the PZT plates should be always placed with its right edge terminating at the beam free end. Moreover, this result is again valid when r varies from 0 to 1 and generalize a result that the authors have demonstrated in [2] for Ω=0. In contrast the position of the left end side changes with the considered modes, ratio r and the angular velocity.

In Fig. 3-6 are reported the optimal configurations when two of the first three modes are excited with different percentage r. The black points show the position of the right end ($a+h/2=L_b$), while the positions of the left end (a-h/2) is represented with different colors for different angular velocities.

The results, when the first two modes are excited, are reported in Fig. 3.

Analyzing the red points (Ω=0) it is possible to notice that, starting from r=0, the optimal first mode configuration (the piezoelectric plates distributed along all of the beam) is retained until r reaches a value of, approximately, 0.5, than it gradually changes to reach the optimal for the second mode in r=1. Moreover when Ω increase, increments the range of r within this optimal first mode configuration is maintained.

A similar behavior is observed also when the first mode is coupled with the third (Fig. 4): in this case the first configuration is held for a bigger range of r and than there is, differently from the previous, a sharp transition (r≅0.78 for Ω=0) to arrive to the optimal configuration in r=1.

When the coupled modes are the second and the third (Fig. 5) there is a difference tendency between Ω=0 and the others. For Ω=0 there is not a range of r where the second optimal mode configuration is held, but there is a constant gradual transition from the second to the third optimal mode configuration. Differently, when there are the effects due to the angular velocity, the second optimal configuration (or similar) is held until r≅0.4 and then it changes.

In all the considered cases it can be noticed that when Ω increases, the shape of the second and third mode changes consequently also the second and third optimal configuration is modified.

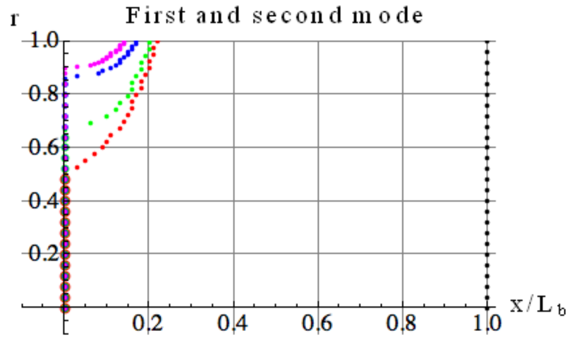

Figure 3. Optimal placement for coupling of the first and second mode (red:Ω=0; green:Ω=3000; blue: Ω=6000; purple: Ω=10000)

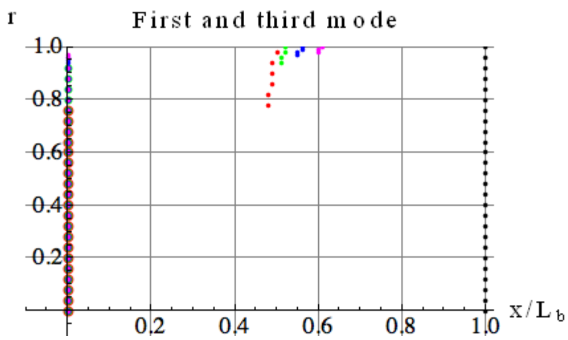

Figure 4. Optimal placement for coupling of the first and third mode (red:Ω=0; green:Ω=3000; blue: Ω=6000; purple: Ω=10000)

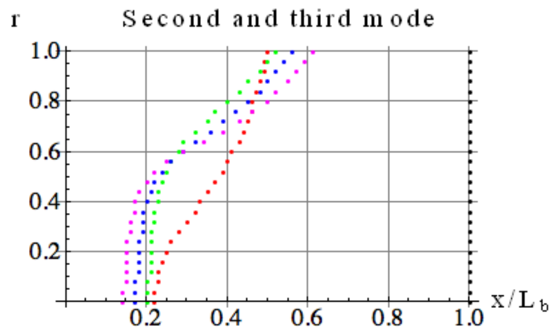

Figure 5. Optimal placement for coupling of the second and third mode (red:Ω=0; green:Ω=3000; blue: Ω=6000; purple: Ω=10000)

4. Conclusions

In this paper the optimal placement of piezoelectric plates to control multimode vibrations of rotating beam has been studied. The first three modes, for different angular velocities and different mode ratio, have been considered and the optimal configurations have been reported. It has been found that the optimal placement for the right end size, independently from the excited modes and for any angular velocity Ω, is at the beam free end. The position of the left end side changes with the considered modes, ratio r and the angular velocity as represented in the figures.

5. References

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